

Closure Report

File No.	MTR/2018/000157
Title of the project	A study on exact generating functions and congruences for partition functions associated to Ramanujan's mock theta functions and other related functions.
Principal Investigator	Prof. Nayandeep Deka Baruah, Tezpur University, Distt. sonitpur p.b.no.72 napaam, tezpur, Tezpur, Assam-784011
Total Released Amount	2,20,000 INR
Date of start of the Project	19 March, 2019
Date of Completion	18 March, 2022
Duration of Project	36 months
Total Expenditure (as on date)	185782

Closure Details

Key outcomes or achievements

Several new exact generating functions and congruences for partition functions associated to Ramanujan's mock theta functions and related functions are found. Some counting theorems of the n -color partition function and relations among representations of integers by certain quadratic forms are also discovered. The contents of our findings have been published in the following 10 papers.

[1] Nayandeep Deka Baruah (with Nilufar Mana Begum): Generating functions and congruences for some partition functions related to mock theta functions, *International Journal of Number Theory*, Vol. 16, No. 2, pp. 423--446, 2020. [2] Nayandeep Deka Baruah (with Mandeep Kaur): A note on some recent results of Da Silva and Sellers on congruences for k -regular partitions with designated summands, *Integers*, Vol. 20, #A74, 6 pp., 2020. [3] Nayandeep Deka Baruah (with Mandeep Kaur): Some results on vanishing coefficients in infinite product expansions, *The Ramanujan Journal*, Vol. 53, No. 3, pp. 551--568, 2020. [4] Nayandeep Deka Baruah (with Hirakjyoti Das): Families of congruences for fractional partition functions modulo powers of primes, *Research in Number Theory*, Vol. 7, Article No. 57, 21 pp., 2021. [5] Nayandeep Deka Baruah (with Hirakjyoti Das): Generating functions and congruences for 9-regular and 27-regular partitions in 3 colors, *Hardy-Ramanujan Journal (Special Commemorative volume in honour of Srinivasa Ramanujan)*, Vol. 44, pp. 101--115, 2021. [6] Nayandeep Deka Baruah (with Subhajit Bandyopadhyay): The n -color partition function and some counting theorems, *Integers*, Vol. 21, #A83, 16 pp., 2021. [7] Nayandeep Deka Baruah (with Hirakjyoti Das): Matching Coefficients in the series expansions of certain q -products and their reciprocals, *The Ramanujan Journal*, 2021, DOI: 10.1007/s11139-021-00534-4. [8] Nayandeep Deka Baruah (with Hirakjyoti Das): Relations among representations of integers by certain quadratic forms, *Indian Journal of Pure and Applied Mathematics*, 2021, DOI: <https://doi.org/10.1007/s13226-021-00158-w>. [9] Nayandeep Deka Baruah (with Hirakjyoti Das): Infinite families of congruences modulo powers of 2 for some partition functions involving only odd parts, *International Journal of Number Theory*, 2022, DOI:10.1142/S1793042122500944 [10] Nayandeep Deka Baruah (with Hirakjyoti Das): On 3^k -regular cubic partitions, *Journal of the Korean Mathematical Society*, 2022, <https://doi.org/10.4134/JKMS.j210517>.

Detailed research report

Recently, Andrews, Dixit and Yee introduced partition functions associated with Ramanujan/Watson third-order mock theta functions $\omega(q)$ and $\nu(q)$. We found several new exact generating functions for those partition functions as well as the associated smallest part functions and deduce several new congruences modulo powers of 5.

We provided additional arithmetic aspects for the number of k -regular partitions with designated summands, denoted by $PD_k(n)$. Using elementary generating function dissections and manipulations, we proved all the conjectural congruences presented by da Silva and Sellers (*Bull. Braz. Math. Soc. (N.S.)* 51 (2020), 357--370). In addition, we provided three new families of congruences modulo 8.

We proved several new results on vanishing coefficients in infinite product expansions.

Recently, Chan and Wang studied the fractional partition function and found several infinite classes of congruences satisfied by the corresponding coefficients. We found new families of congruences modulo powers of primes using the Rogers-Ramanujan continued fraction and some dissection formulae of certain q -products. We also found analogous congruences in the coefficients of the fractional powers of the generating function for the 2-color partition function.

We found exact generating functions and new infinite classes of congruences for 9-regular and 27-regular partitions in 3 colours as well as 3^k -regular cubic partitions.

We defined a new counting function on the set of n -color partitions of a nonnegative integer and related the function with the n -color partition function and other well-known arithmetic functions like the Mobius function, Liouville function, etc. and their divisor sums. Furthermore, we use a counting method of Erdos to obtain some counting theorems for n -color partitions that are analogous to those found by Andrews and Deutsch for the ordinary partition function.

We showed that the series expansions of certain q -products have matching coefficients with their reciprocals. Several of the results are associated to some of Ramanujan's famous continued fractions.

We found interesting relations among representations of integers by certain quadratic forms.

We found infinite families of congruences modulo powers of 2 for some partition functions involving only odd parts.

Number of students/researchers trained

List of Publications (only from SCI indexed journals)

Title of the Paper	List of Authors	Name of Journal	Type of Journal	Status
Infinite families of congruences modulo powers of 2 for some partition functions involving only odd parts	Nayandeep Deka Baruah and Hirakjyoti Das	International Journal of Number Theory	International	Published
On 3^k-regular cubic partitions	Nayandeep Deka Baruah and Hirakjyoti Das	JOURNAL OF THE KOREAN MATHEMATICAL SOCIETY	International	Published
Matching coefficients in the series expansions of certain q-products and their reciprocals	Nayandeep Deka Baruah and Hirakjyoti Das	RAMANUJAN JOURNAL	International	Published
Families of congruences for fractional partition functions modulo powers of primes	Nayandeep Deka Baruah and Hirakjyoti Das	Research in Number Theory	International	Published
Relations among representations of integers by certain quadratic forms	Nayandeep Deka Baruah and Hirakjyoti Das	Indian Journal of Pure and Applied Mathematics	International	Published
Some results on vanishing coefficients in infinite product expansions.	Nayandeep Deka Baruah and Mandep Kaur	RAMANUJAN JOURNAL	International	Published
Generating functions and congruences for some partition functions related to mock theta functions	Nayandeep Deka Baruah and Nilufar Mana Begum	International Journal of Number Theory	International	Published

List of Papers Published in Conference Proceedings, Popular Journals

Title of the Paper	List of Authors	Name of Journal	Type of Journal	Status
Generating functions and congruences for 9-regular and 27-regular partitions in 3 colours	Nayandeep Deka Baruah and Hirakjyoti Das	Hardy-Ramanujan Journal	International	Published
THE n-COLOR PARTITION FUNCTION AND SOME COUNTING THEOREMS	Nayandeep Deka Baruah and Subhjit Bandyopadhyay	Integers	International	Published
A NOTE ON SOME RECENT RESULTS OF DA SILVA AND SELLERS ON CONGRUENCES FOR k-REGULAR PARTITIONS WITH DESIGNATED SUMMANDS	Nayandeep Deka Baruah and Mandep Kaur	Integers	International	Published

List of Patents filed/ to be filed

No Such Record Found

Collaborative visits in India and abroad using MATRICS grant

No Such Record Found

Any other collaborative work carried out

RECURRING
GFR 12 – A
[(See Rule 238 (1))]
UTILIZATION CERTIFICATE (UC) FOR THE YEAR 2021-22
in respect of RECURRING
as on 31st March 2022 to be submitted to SERB
UC (Provisional/Audited)
(To be given separately for each financial year ending on 31st March)

1. Name of the grant receiving Organization : **Tezpur University**
2. Name of Principal Investigator (PI) : **Dr. Nayandeep Deka Baruah**
3. SERB Sanction order no. & date : No. MTR/2018/000157 dated 15-03-2019
4. Title of the Project : A study on exact generating functions and congruences for partition functions associated to Ramanujan's mock theta functions and other related functions.
5. Name of the SERB Scheme : **MTR – MATRICS**
6. Whether recurring or non-recurring grants : **Recurring**
7. Grants position at the beginning of the financial year (Grants released by SERB)
 - (i) Cash in Hand/Bank /Carry forward from previous financial year : 60,763/-
 - (ii) Others, If any : **NIL**
 - (iii) Total : 60,763/-

Unspent Balance of Grants received previous years [figure as at Sl.No. 7(iii)]	Interest Earned thereon	Interest deposited back to the SERB	Grants received during the year			Total Available funds (1+2-3+4)	Expenditure incurred	Closing Balances (5-6)	Remark
			Sanction No. (i)	Date (ii)	Amount (iii)				
1	2	3	4			5	6	7	8
60,763/-	893/-	Nil	Nil	Nil	Nil	61,656/-	25,063/-	36,593/-	

8. Details of grants received, expenditure incurred and closing balances: (Actuals)

9. Component wise utilization of grants

Grants-in-aid- General	Total	Remark
Equipment	0	
Travel	0	
Research Grant	25,063/-	
Overhead	0	
GRAND TOTAL	25,063/-	

10. Details of grants position at the end of the year

- (I) Cash in Hand/Bank : **Rs. 36,593/-**
- (II) Refunds to SERB, If any (to be refunded) : **Rs. 36,593/-**
- (III) Balance (Completion of the project): **Nil**

Nayandeep Deka Baruah

GFR 12 – A

[(See Rule 238 (1))]

UTILIZATION CERTIFICATE (UC) FOR THE YEAR 2021-22

in respect of RECURRING

as on 31st March 2022 to be submitted to SERB

Is the UC (Provisional/Audited)

(To be given separately for each financial year ending on 31st March)

Certified that I have satisfied that the conditions on which grants were sanctioned have been duly fulfilled/are being fulfilled and that I have exercised following checks to see that the money has been actually utilized for the purpose for which it was sanctioned:

- (i) The main accounts and other subsidiary accounts and registers (including assets registers) are maintained as prescribed in the relevant Act/Rules/Standing instructions (mention the Act/Rules) and have been duly audited by designated auditors. The figures depicted above tally with the audited figures mentioned in financial statements/accounts.
- (ii) There exist internal controls for safeguarding public funds/assets, watching outcomes and achievements of physical targets against the financial inputs, ensuring quality in asset creation etc. & the periodic evaluation of internal controls is exercised to ensure their effectiveness.
- (iii) To the best of our knowledge and belief, no transactions have been entered that are in violation of relevant Act/Rules/standing instructions and scheme guidelines.
- (iv) The responsibilities among the key functionaries for execution of the scheme have been assigned in clear terms and are not general in nature.
- (v) The benefits were extended to the intended beneficiaries and only such areas/districts were covered where the scheme was intended to operate.
- (vi) The expenditure on various components of the scheme was in the proportions authorized as per the scheme guidelines and terms and conditions of the grants-in-aid.
- (vii) It has been ensured that the physical and financial performance under
(CRG/NPDF/ECR.....etc.) (Name of the scheme has been according to the requirements, as prescribed in the guidelines issued by Govt. of India and the performance/targets achieved statement for the year to which the utilization of the fund resulted in outcomes given at Annexure

I duly enclosed.

- (viii) The utilization of the fund resulted in outcomes given at Annexure – II duly enclosed (to be formulated by the Ministry/Department concerned as per their requirements/specifications.)
- (ix) Details of various schemes executed by the agency through grants-in-aid received from the same Ministry or from other Ministries is enclosed at Annexure –II (to be formulated by the Ministry/Department concerned as per their requirements/specifications).

Date: April 18, 2022

Place: Naipoom, Tezpur

Signature Name: Prof. N. D. Baruah Principal Investigator(PI)	Signature with Seal Name: Finance Officer Tezpur University	Signature with Seal Name: Registrar Tezpur University
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भारतीय स्टेट बैंक
State Bank of India
Issuing Branch: TEZPUR UNIVERSITY
कोड क्र. / CODE No: 14259
Tel No. 03712-267285

A/C Payee

मांगद्राफ्ट
DEMAND DRAFT

Key: VUDBUT
Sr. No: 162501

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D D M M Y Y Y Y

मांगे जानेपर FUND FOR SCIENCE AND ENGINEERING RESEARCH*****

या उनके आदेश पर
OR ORDER

ON DEMAND PAY

रुपये RUPEES Thirty Six Thousand Five Hundred and Ninety Three Only

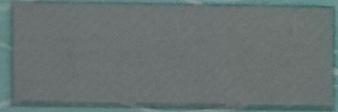
अदा करें ₹ 36593.00

IOI 000536846757
Name of Applicant

Key: VUDBUT Sr. No: 162591
TEZPUR UNIVERSITY R & D

AMOUNT BELOW 36594(0/5)

मूल्य प्राप्त / VALUE RECEIVED



Satyajit Mazumdar
शाखा प्रबंधक
BRANCH MANAGER

भारतीय स्टेट बैंक
STATE BANK OF INDIA
अवकर्ता शाखा / DRAWEE BRANCH: NEW DELHI MAIN BRANCH
कोड क्र. / CODE No: 00691

प्राधिकृत हस्ताक्षरकर्ता
AUTHORISED SIGNATORY

कम्प्यूटर द्वारा मुद्रित होने पर ही वैध
VALID ONLY IF COMPUTER PRINTED

केवल 3 महीने के लिए वैध
VALID FOR 3 MONTHS ONLY

₹ 1,50,000/- एवं अधिक के लिखत दो अधिकारियों द्वारा हस्ताक्षरित होने पर ही वैध है।
INSTRUMENTS FOR ₹ 1,50,000/- & ABOVE ARE NOT VALID UNLESS SIGNED BY TWO OFFICERS.

⑈846757⑈ 000002000⑈ 000536⑈ 16

सत्यजीत मजूमदार
SATYAJIT MAZUMDAR
M 13712

SESHAASA(D) / CTS-2010

Closure Report

File Number : MTR/2018/000157

Project Title : A study on exact generating functions and congruences for partition functions associated to Ramanujan's mock theta functions and other related functions.

Principal Investigator : [Prof. Nayandeep Deka Baruah](#)
Tezpur University
Distt. sonitpur p.b.no.72 napaam, tezpur, Tezpur, Assam-784011

Total Released Amount : 2,20,000 (INR)

Start Date of the Project: 19 Mar, 2019

Date of completion: 18 Mar, 2022 (36 months)

Total Expenditure : 1,85,782 (INR)

Closure Details

Key outcomes or achievements

Several new exact generating functions and congruences for partition functions associated to Ramanujan's mock theta functions and related functions are found. Some counting theorems of the n -color partition function and relations among representations of integers by certain quadratic forms are also discovered. The contents of our findings have been published in the following 10 papers. [1] Nayandeep Deka Baruah (with Nilufar Mana Begum): Generating functions and congruences for some partition functions related to mock theta functions, *International Journal of Number Theory*, Vol. 16, No. 2, pp. 423--446, 2020. [2] Nayandeep Deka Baruah (with Mandeep Kaur): A note on some recent results of Da Silva and Sellers on congruences for k -regular partitions with designated summands, *Integers*, Vol. 20, #A74, 6 pp., 2020. [3] Nayandeep Deka Baruah (with Mandeep Kaur): Some results on vanishing coefficients in infinite product expansions, *The Ramanujan Journal*, Vol. 53, No. 3, pp. 551--568, 2020. [4] Nayandeep Deka Baruah (with Hirakjyoti Das): Families of congruences for fractional partition functions modulo powers of primes, *Research in Number Theory*, Vol. 7, Article No. 57, 21 pp., 2021. [5] Nayandeep Deka Baruah (with Hirakjyoti Das): Generating functions and congruences for 9-regular and 27-regular partitions in 3 colors, *Hardy-Ramanujan Journal (Special Commemorative volume in honour of Srinivasa Ramanujan)*, Vol. 44, pp. 101--115, 2021. [6] Nayandeep Deka Baruah (with Subhajit Bandyopadhyay): The n -color partition function and some counting theorems, *Integers*, Vol. 21, #A83, 16 pp., 2021. [7] Nayandeep Deka Baruah (with Hirakjyoti Das): Matching Coefficients in the series expansions of certain q -products and their reciprocals, *The Ramanujan Journal*, 2021, DOI: 10.1007/s11139-021-00534-4. [8] Nayandeep Deka Baruah (with Hirakjyoti Das): Relations among representations of integers by certain quadratic forms, *Indian Journal of Pure and Applied Mathematics*, 2021, DOI: <https://doi.org/10.1007/s13226-021-00158-w>. [9] Nayandeep Deka Baruah (with Hirakjyoti Das): Infinite families of congruences modulo powers of 2 for some partition functions involving only odd parts, *International Journal of Number Theory*, 2022, DOI:10.1142/S1793042122500944 [10] Nayandeep Deka Baruah (with Hirakjyoti Das): On 3^k -regular cubic partitions, *Journal of the Korean Mathematical Society*, 2022, <https://doi.org/10.4134/JKMS.j210517>.

Detailed research report

Recently, Andrews, Dixit and Yee introduced partition functions associated with Ramanujan/Watson third-order mock theta functions (q) and (q) . We found several new exact generating functions for those partition functions as well as the associated smallest part functions and deduce several new congruences modulo powers of 5. We provided additional arithmetic aspects for the number of k -regular partitions with designated summands, denoted by $PD_k(n)$. Using elementary generating function dissections and manipulations, we proved all the conjectural congruences presented by da Silva and Sellers (*Bull. Braz. Math. Soc. (N.S.)* 51 (2020), 357--370). In addition, we provided three new families of congruences modulo 8. We proved several new results on vanishing coefficients in infinite product expansions. Recently, Chan and Wang studied the fractional partition function and found several infinite classes of congruences satisfied by the corresponding coefficients. We found new families of congruences modulo powers of primes using the Rogers-Ramanujan continued fraction and some dissection formulae of certain q -products. We also found analogous congruences in the coefficients of the fractional powers of the generating function for the 2-color partition function. We found exact generating functions and new infinite classes of congruences for 9-regular and 27-regular partitions in 3 colours as well as 3^k -regular cubic partitions. We defined a new counting function on the set of n -color partitions of a nonnegative integer and related the function with the n -color partition function and other well-known arithmetic functions like the Mobius function, Liouville function, etc. and their divisor sums. Furthermore, we use a counting method of Erdos to obtain some counting theorems for n -color partitions that are analogous to those found by Andrews and Deutsch for the ordinary partition function. We showed that the series expansions of certain q -products have matching coefficients with their reciprocals. Several of the results are associated to some of Ramanujan's famous continued fractions. We found interesting relations among representations of integers by certain quadratic forms. We found infinite families of congruences modulo powers of 2 for some partition functions involving only odd parts.

Number of students/Researchers trained : 4

List of Publications (only from SCI indexed journals) :

Title of the Paper	List of Authors	Journal Details	Month & Year	Volume	Status	DOI No	Impact Factor
Relations among representations of integers by certain quadratic forms	Nayandeep Deka Baruah and Hirakjyoti Das	Indian Journal of Pure and Applied Mathematics (International)	Aug-2021	Online First (1--11)	Published	https://doi.org/10.1007/s13226-021-00158-w	0.372
Generating functions and congruences for some partition functions related to mock theta functions	Nayandeep Deka Baruah and Nilufar Mana Begum	International Journal of Number Theory (International)	Mar-2020	16 (423-446)	Published	DOI: 10.1142/S1793042120500220	0.674
Infinite families of congruences modulo powers of 2 for some partition functions involving only odd parts	Nayandeep Deka Baruah and Hirakjyoti Das	International Journal of Number Theory (International)	Apr-2022	Online Ready (1--20)	Published	DOI: 10.1142/S1793042122500944	0.674
On 3^k -regular cubic partitions	Nayandeep Deka Baruah and Hirakjyoti Das	JOURNAL OF THE KOREAN MATHEMATICAL SOCIETY (International)	Feb-2022	Ahead of Print Article (1--13)	Published	https://doi.org/10.4134/JKM.S.j210517	0.583
Some results on vanishing coefficients in infinite product expansions.	Nayandeep Deka Baruah and Mandeep Kaur	RAMANUJAN JOURNAL (International)	Dec-2020	53 (551--568)	Published	https://doi.org/10.1007/s11139-019-00172-x	0.837
Matching coefficients in the series expansions of certain q -products and their reciprocals	Nayandeep Deka Baruah and Hirakjyoti Das	RAMANUJAN JOURNAL (International)	Jan-2022	Online First (1--38)	Published	https://doi.org/10.1007/s11139-021-00534-4	0.837
Families of congruences for fractional partition functions modulo powers of primes	Nayandeep Deka Baruah and Hirakjyoti Das	Research in Number Theory (International)	Dec-2021	7 (1-21)	Published	https://doi.org/10.1007/s40993-021-00287-5	Not yet assigned

List of Papers Published in Conference Proceedings, Popular Journals :

Title of the Paper	List of Authors	Journal Details	Month & Year	Volume	Status	DOI No	Impact Factor
Generating functions and congruences for 9-regular and 27-regular partitions in 3 colours	Nayandeep Deka Baruah and Hirakjyoti Das	Hardy-Ramanujan Journal (International)	Dec-2021	44 (102--115)	Published	https://doi.org/10.46298/hrj.2022.8927	
A NOTE ON SOME RECENT RESULTS OF DA SILVA AND SELLERS ON CONGRUENCES FOR K -REGULAR PARTITIONS WITH DESIGNATED SUMMANDS	Nayandeep Deka Baruah and Mandeep Kaur	Integers (International)	Sep-2020	20 (1--6)	Published	http://math.colgate.edu/~integers/u74/u74.pdf	
THE n -COLOR PARTITION FUNCTION AND SOME COUNTING THEOREMS	Nayandeep Deka Baruah and Subhajit Bandyopadhyay	Integers (International)	Aug-2021	21 (1--16)	Published	http://math.colgate.edu/~integers/v83/v83.pdf	

List of Patents filed/ to be filed :

Patent Title	Authors	Patent Type	Country/Agency Name	Patent Status	Application/Grant No.
Not Available					

Collaborative visits in India and abroad using MATRICS grant

Any other collaborative work carried out

All published papers are written jointly with my Ph.D. students.

**REQUEST FOR ANNUAL INSTALMENT WITH UP-TO-DATE STATEMENT OF
EXPENDITURE(SOE)**

1. Sanction Order No and date: No. MTR/2018/000157 dated 15-03-2019
2. Total Project Cost: Rs. 6,60,000/-
3. Revised Project Cost: (if applicable) Nil
4. Date of Commencement: 19th March 2019
5. Statement of Expenditure: Month wise expenditure incurred during the financial year 2021-22

Month & year	Expenditure incurred/ committed	Remark
April, 2020	NIL	
May, 2020	NIL	
June, 2020	NIL	
July, 2020	NIL	
August, 2020	NIL	
September, 2020	NIL	
October, 2020	NIL	
November, 2020	NIL	
December, 2020	NIL	
January, 2021	NIL	
February, 2021	NIL	
March, 2021	25,063.00	
Total	25,063.00	

6. Grant received in each year:

Sl No.	Particulars	Amount (in Rs.)
a.	1 st Year	2,20,000/-
b.	2 nd Year	Nil
c.	3 rd year	Nil
d.	Interest if any (during 2020-21 & 2021-22)	1482.00 +893.00
Total(a+b+c+d)		2,22,375.00

[Handwritten Signature]

SE for Recurring Grant

Scheme: MATRICS

Annexure-III

STATEMENT OF EXPENDITURE(SOE) (1st April 2021 to 31st March 2022)

Sr. No.	Sanctioned Heads	Grant received till date	Expenditure Incurred				Total Expenditure till 31 st March 2022	Balance as on 1 st April 2022	Remarks (if any)
			1 st Year (19 th March 2019 to 31 st March 2019)	2 nd Year (1 st April 2019 to 31 st March 2020)	3 rd Year (1 st April 2020 to 31 st March 2021)	4 th Year (1 st April 2021 to 31 st March 2022)			
(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII = IV + V + VI + VII)	(IX = III - VIII)	
1.	Research Grant	2,00,000/-	0	87,165/-	61,054/-	25,063/-	173,282/-	26,718/-	Completion of the project. Amount to be refunded to SERB = Rs. 36,593/-
2	Overhead Expenses	20,000/-	0	12,500/-	0		12,500/-	7,500/-	
	Total	2,20,000/-	0	99,665/-	61,054/-	25,063/-	1,85,782/-	34,218/-	
3.	Interest earned	2,375/- (1482+893)	-	-	0		-	2,375/-	
	Grand Total	2,22,375/-	-	99,665/-	61,054/-	25,063/-	1,85,782/-	36,593/-	

Signature

Name: Prof. N. D. Baruah
Principal Investigator (PI)

Signature with Seal

Name:
Finance OfficerFinance Officer
Tezpur University

Signature with Seal

Name:
Head of OrganisationRegistrar
Tezpur University