

UNIVERSITY GRANTS COMMISSION  
BAHADUR SHAH ZAFAR MARG  
NEW DELHI - 110002

Annual Report of the works done on the Major Research Project

1. Project Report no.: 2<sup>nd</sup>.
2. UGC reference no.: UGC Approval No. F. 37-369/2009 (SR), Ref. Lett. no. F.No.37-1/2009(ASS)(SR) dated. 12-01-2010.
3. Period of report: From 1<sup>st</sup> April, 2011 to 31<sup>st</sup> March, 2012.
4. Title of research project: "Studies on Gluon Distribution Function and Recombination of Partons".
5. (a) Name of the Principal Investigator: Prof. J. K. Sarma.  
(b) Deptt. and University where work has progressed: Department of Physics, Tezpur University.
6. Date of joining of the Project Fellow: 16.07.2010.
7. Effective date of starting of the project: 16.07.2010 (considering the date of joining of the project fellow).
8. Report of the works done: Attached.

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25/7/2011

**UNIVERSITY GRANTS COMMISSION  
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**Report of the Progress during the last financial year April, 2011 – March, 2012 of the Major Research Project “Studies on Gluon Distribution Function and Recombination of Partons” with UGC’S letter F. No. 37-1/2009 (ASS)(SR) dated 12 January 2010**

1. Completed the phenomenological study of the solution of linear DGLAP equation for singlet structure function in LO, NLO and NNLO respectively.
2. Attended the DAE-BRNS workshop on Hadron Physics.
3. Bought different books related to the project.
4. Solved the nonlinear GLR evolution equation and obtained the t-evolution of the gluon distribution function.

**Buying of the books:**

According to our scheme submitted for the project and following university procedure we bought three books related to our work. The lists of the books are as follows:

- (i) “QCD as a Theory of Hadrons” by Stephan Narison.
- (ii) “Deep Inelastic Scattering” by R. Devenish and A. Cooper-Sarkar.
- (iii) “Foundations of Perturbative QCD” by John Collins.

**Research Work :**

**A. A phenomenological study of the solution of DGLAP evolution equation for Singlet structure function upto NNLO:**

The DGLAP evolution equation for singlet structure function in NNLO is

$$\frac{\partial F_2^S}{\partial \alpha} - \frac{\alpha_S(t)}{2\pi} \left[ \frac{2}{3} (3 + 4 \ln(1-x)) F_2^S(x,t) + I_1^S(x,t) \right] - \left( \frac{\alpha_S(t)}{2\pi} \right)^2 I_2^S(x,t) - \left( \frac{\alpha_S(t)}{2\pi} \right)^3 I_3^S(x,t) = 0, \quad (1)$$

where

$$I_1^S(x, t) = \frac{4}{3} \int \frac{d\omega}{x(1-\omega)} \left[ (1+\omega^2) F_2^S\left(\frac{x}{\omega}, t\right) - 2F_2^S(x, t) \right] + N_f \int \frac{1}{x} \left\{ \omega^2 + (1-\omega)^2 \right\} G\left(\frac{x}{\omega}, t\right) d\omega,$$

$$I_2^S = \left[ (x-1) F_2^S(x, t) \int_0^1 f(\omega) d\omega + \int_x^1 f(\omega) F_2^S\left(\frac{x}{\omega}, t\right) d\omega + \int_x^1 F_{qq}^S(\omega) F_2^S\left(\frac{x}{\omega}, t\right) d\omega + \int_x^1 F_{qg}^S(\omega) G\left(\frac{x}{\omega}, t\right) d\omega \right],$$

$$I_3^S(x, t) = \int \frac{d\omega}{x} \frac{1}{w} \left[ P_{qq}(x) F_2^{NS}\left(\frac{x}{\omega}, t\right) + P_{qg}(x) G\left(\frac{x}{w}, t\right) \right].$$

For simplicity, we can assume  $G(x, t) = K(x) F_2^S(x, t)$ , where  $K(x)$  is a suitable function of  $x$  or may be a constant. Now, substituting all these, and using Taylor's expansion method and performing  $u$  integrations we get from equation (1)

$$-t \frac{\partial F_2^S(x, t)}{\partial t} + L(x, t) \frac{\partial F_2^S(x, t)}{\partial x} + M(x, t) F_2^S(x, t) = 0, \quad (2)$$

where  $L(x, t)$  and  $M(x, t)$  are functions of  $x$  and  $t$ . Let us introduce the functions  $\bar{L}(x)$  and  $\bar{M}(x)$  such that  $L(x, t) = \frac{\beta_0 t}{2} T(t) \bar{L}(x)$  and  $M(x, t) = \frac{\beta_0 t}{2} T(t) \bar{M}(x)$ . Thus solving equation (2) we obtain

$$F_2^S(x, t) = -\gamma t^{1+\left(\frac{b-b^2}{t}\right)} \cdot \exp\left(\frac{b-c-b^2 \ln^2 t}{t}\right) \exp\left[\int \left(\frac{1}{L(x)} - \frac{\bar{M}(x)}{L(x)}\right) dx\right], \quad (3)$$

where  $\gamma$ ,  $b$  and  $c$  are some constants. Now defining an input point  $F_2^S(x, t_0)$  at  $t = t_0$ , where  $t_0 = \ln(Q^2/\Lambda^2)$  for any lower value  $Q^2 = Q_0^2$ , we get the  $t$ -evolution of  $F_2^S(x, t)$  structure function in NNLO from equation (3) as

$$F_2^S(x, t) = F_2^S(x, t_0) \frac{t^{1+(b-b^2)/t}}{t_0^{1+(b-b^2)/t_0}} \cdot \exp\left\{\left(\frac{b-c-b^2 \ln^2 t}{t}\right) / \frac{b-c-b^2 \ln^2 t_0}{t_0}\right\}. \quad (4)$$

Similarly defining  $F_2^S(x_0, t)$  at  $x = x_0$ , the  $x$ -evolution of singlet structure function in NNLO is obtained from equation (3) as

$$F_2^S(x, t) = F_2^S(x_0, t) \exp\left[\int_{x_0}^x \left(\frac{1}{L(x)} - \frac{\bar{M}(x)}{L(x)}\right) dx\right]. \quad (5)$$

As the deuteron structure function is relation to the singlet structure function as  $F_2^d(x, t) = \frac{5}{9}F_2^S(x, t)$ , so we obtain the  $t$  and  $x$ -evolution of deuteron structure function as

$$F_2^d(x, t) = F_2^d(x, t_0) \left( \frac{t^{1+(b-b^2)/t}}{t_0^{1+(b-b^2)/t_0}} \right) \exp \left( \frac{b-c-b^2 \ln^2 t}{t} - \frac{b-c-b^2 \ln^2 t_0}{t_0} \right) \quad (6)$$

$$F_2^d(x, t) = F_2^d(x_0, t) \exp \left[ \int_{x_0}^x \left( \frac{1}{\bar{L}(x)} - \frac{\bar{M}(x)}{\bar{L}(x)} \right) dx \right] \quad (7)$$

Equations (6) and (7) are used in our phenomenological work for calculation of deuteron structure function. We have compared our results of  $t$  and  $x$ -evolutions of deuteron structure function with E665 and NMC experimental data, and NNPDF parameterization. We consider the range  $0.0052 \leq x \leq 0.18$  and  $1.094 \leq Q^2 \leq 26 \text{ GeV}^2$  for E665 data,  $0.0045 \leq x \leq 0.18$  and  $0.75 \leq Q^2 \leq 27 \text{ GeV}^2$  for NMC data and  $0.0045 \leq x \leq 0.09$  and  $1.25 \leq Q^2 \leq 26 \text{ GeV}^2$  for NNPDF parameterization. In the  $t$ -evolutions of deuteron structure function our computed values of from equation (6) are plotted against  $Q^2$  for different values of  $x$ . On the other hand, for  $x$ -evolutions our computed values of from equation (7) are plotted against  $x$  for different values of  $Q^2$ . We observe that the  $t$  and  $x$ -evolutions of deuteron structure function are in good consistency with the experimental data and parametrization.

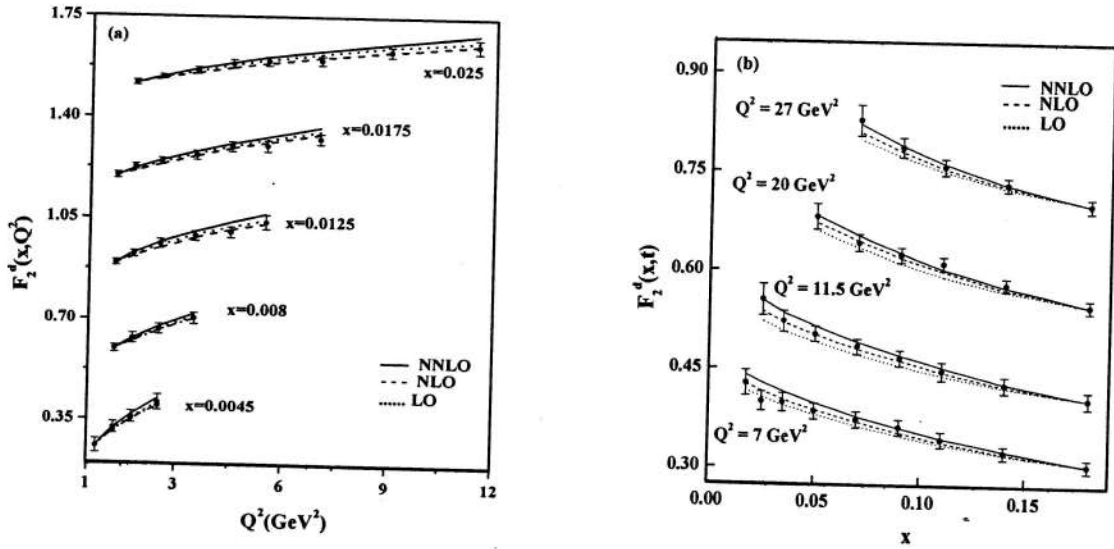


Fig. 1 (a, b) NMC data: Dotted lines are LO results, dashed lines are NLO and solid lines are our NNLO results. For clarity, data are scaled up by  $+0.2i$  (in Fig. 1(a)) and  $+i$  (in Fig.1(b)) (with  $i = 0,1,2,3$ ) starting from the bottom of all graphs in each figure.

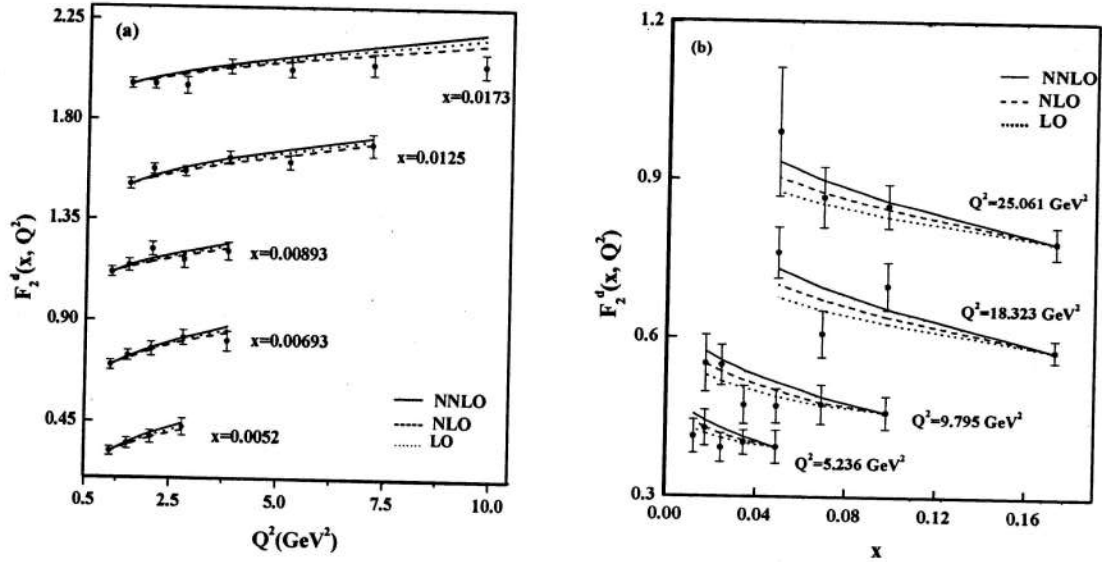


Fig. 2 (a, b) E665 data: Dotted lines are LO results, dashed lines are NLO and solid lines are our NNLO results. For clarity, data are scaled up by +0.4i (in Fig. 2(a)) and +0.1i (in Fig.2(b)) (with  $i = 0,1,2,3$ ) starting from the bottom of all graphs in each figure.

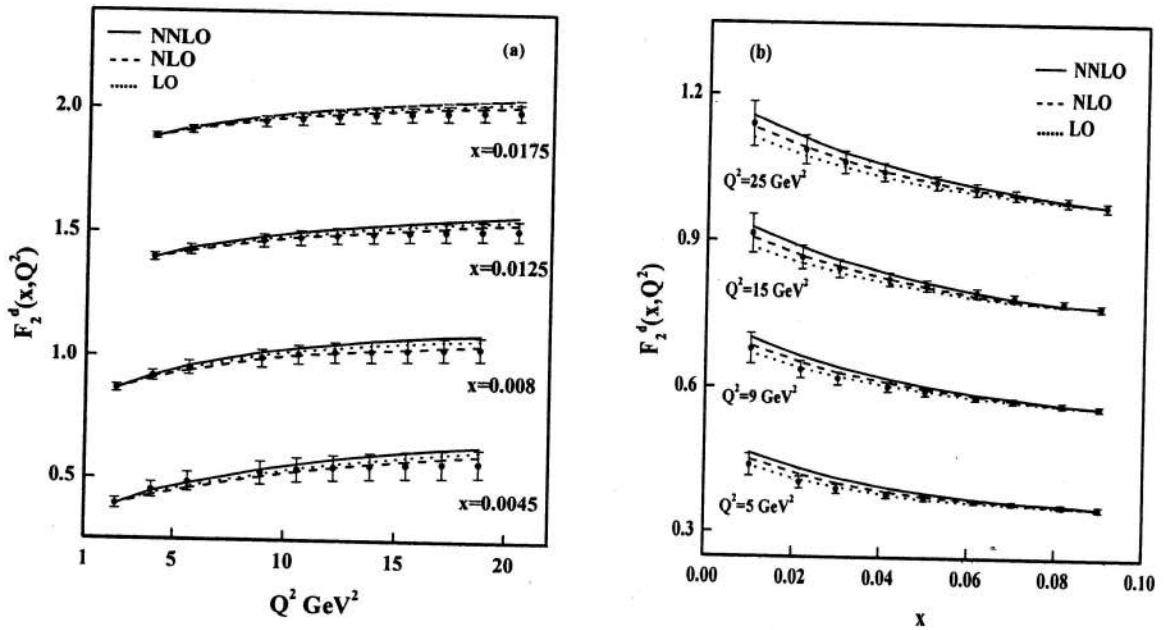


Fig. 3. (a, b) NNPDF data: Dotted lines are LO results, dashed lines are NLO and solid lines are our NNLO results. For clarity, data are scaled up by +0.4i (in Fig. 3(a)) and +0.1i (in Fig.3(b)) (with  $i = 0,1,2,3$ ) starting from the bottom of all graphs in each figure.

## **B. Solution of Nonlinear GLR evolution equation for gluon distribution function:**

The small- $x$  limit of parton (quarks and gluons) distribution is of considerable importance both theoretically and phenomenologically. Detailed theoretical QCD studies have been made to predict the parton distribution at small- $x$ . These theoretical expectations must be incorporated in any phenomenological analysis of structure function data. Such analysis is expected to give realistic extrapolation of the parton distribution into the small  $x$  regime. The determination of the gluon density is of great importance at low  $x$ , where gluons are expected to be dominant. As  $x$  decreases for fixed  $Q$ , the number of partons increases, and at some value of  $x = x_{cr}$  partons start to populate the whole hadron disc densely. For  $x < x_{cr}$  the partons overlap spatially and begin to interact throughout the disc. The increase in the number of small- $x$  partons becomes limited by gluon fusion ( $gg \rightarrow g$ ). So, any further increase in the energy will not increase the probed parton density. The value of  $Q^2$  at which this happens is known as saturation scale  $Q_{sat}^2$ . It is also important to know the value of  $Q_{sat}^2$  at which parton saturation occurs. The region where the partons get saturated is called the black disc and this blackening of proton is called the saturation or shadowing.

Partonic saturation plays also an important role in the context of the unitarity bound. It is well known that the hadronic cross sections obey the Froissart bound which stems from the general assumptions of the analyticity and unitarity of the scattering amplitude. The Froissart bound implies that the total cross section does not grow faster than the logarithm squared of the energy

$$\sigma_{tot} \leq \frac{\pi}{m_{\pi}^2} (\ln s)^2$$

where  $m_{\pi}$  is the mass of pion and it determines the scale of the range of the strong force. It is generally believed, that the parton saturation mechanism leads to the unitarization of the cross section at high energies.

The correlation among the initial partons are neglected in the derivation of linear DGLAP evolution equation, where only parton splitting is considered. But at small- $x$  the corrections of the correlations among initial gluons to the elementary amplitude should also be considered.

Therefore at small- $x$  apart from the production diagram one has to include the additional diagrams which take into account gluon recombination. This leads to the modification of the evolution equation by a term which is non-linear in gluon density, i.e., in the small- $x$  limit, linear DGLAP evolution equation are expected to breakdown and the evolution is given by a non-linear evolution equation. Gribov-Levin-Reskin first studied these non-linear effects.

Now the number of gluons  $\rho$  per unit of rapidity which can interact with the probe is  $xg(x, Q^2)$ . If we note that the gluon-gluon cross-section is  $\sigma_{gg \rightarrow g} \approx \alpha_s / Q^2$ , then, when, with decreasing  $x$ , the area  $\rho \sigma_{gg \rightarrow g}$  becomes significant in comparison to  $\pi R^2$  (where  $R$  is the radius of the proton where the gluons populated), shadowing corrections can no longer be neglected. Clearly the crucial parameter is

$$W = \frac{\rho}{\pi R^2} \sigma_{gg \rightarrow g} = \frac{\alpha_s}{Q^2} \frac{xg(x, Q^2)}{\pi R^2}$$

In the region of  $x$  and  $Q^2$  where  $W \ll 1$ , the interaction of gluons (in different cascade ladders) is negligible and we may continue to use evolution equations linear in  $G(x, Q^2)$ . However, at small  $x$  when  $W = \alpha_s$  two gluons in different cascades may interact, generally fusing the gluon ladders together and so decrease the gluon density. This leads to parton recombination or parton saturation or shadowing corrections. Shadowing corrections modify the evolution equations by the addition of a term proportional to  $W$  and quadratic in  $G(x, Q^2)$  which is given as

$$\begin{aligned} \frac{dG(x, Q^2)}{d \ln Q^2} &= \frac{\alpha_s}{2\pi} \int_0^{1-x} dz G\left(\frac{x}{1-z}, Q^2\right) P_{gg}\left(\frac{1}{1-z}\right) - \frac{81 \alpha_s^2}{16 R^2 Q^2} \int_0^{1-x} \frac{dz}{1-z} \left[ G\left(\frac{x}{1-z}, Q^2\right) \right]^2 \\ &= \frac{3\alpha_s}{\pi} \int_0^{\frac{1-x}{x_0}} dz G\left(\frac{x}{1-z}, Q^2\right) \left( \frac{1}{1-z} \right) - \frac{81 \alpha_s^2}{16 R^2 Q^2} \int_0^{\frac{1-x}{x_0}} \frac{dz}{1-z} \left[ G\left(\frac{x}{1-z}, Q^2\right) \right]^2 \end{aligned} \quad (1)$$

This equation is known as the GLR evolution equation. Here  $G(x, Q^2)$  is the gluon distribution function,  $P_{gg}$  is the gluon-gluon splitting function and  $\chi = \frac{x}{x_0}$ , where  $x_0$  is the boundary condition that the gluon distribution joints smoothly onto the unshadowed region. Here

the quark gluon emission diagrams are neglected due to their little importance in the gluon-rich small-x region. Practically,  $x_0 \approx 10^{-2}$ . At  $x=x_0$  the shadowing and unshadowing gluon distribution behaviours are equal whereas, at  $x \ll x_0$  all typical features of small-x physics should be seen. The negative sign in front of the nonlinear term is responsible for the gluon recombination. The strong growth generated by the linear DLLA term is damped whenever gluon density  $G(x, Q^2)$  becomes large, of the order  $1/\alpha_s$ .

Now to solve the GLR equation, let us consider a simple form of Regge behaviour for the determination of the gluon distribution function with the shadowing correction as

$$\begin{aligned} G\left(\frac{x}{1-z}, Q^2\right) &= T(Q^2) \left(\frac{x}{1-z}\right)^{-\lambda} \\ &= T(Q^2) x^{-\lambda} (1-z)^\lambda \\ &= G(x, Q^2) (1-z)^\lambda \end{aligned}$$

where,  $\lambda$  = Pomeron intercept. Here we consider  $\lambda \approx 0.5$  at small-x.

We consider a variable  $\omega$ , so that  $1-z=\omega$ .

Since,  $0 < z < 1 - \frac{x}{x_0}$  so,  $1 < \omega < \frac{x}{x_0}$ .

Thus,  $G\left(\frac{x}{\omega}, Q^2\right) = G(x, Q^2) \omega^\lambda$ .

Substituting all these in equation (1), the GLR equation becomes

$$\begin{aligned} \frac{dG(x, Q^2)}{d \ln Q^2} &= \frac{3\alpha_s}{\pi} \int_1^{\frac{x}{x_0}} G\left(\frac{x}{\omega}, Q^2\right) \left(\frac{-d\omega}{\omega}\right) - \frac{81\alpha_s^2}{16R^2 Q^2} \int_1^{\frac{x}{x_0}} \left[G\left(\frac{x}{\omega}, Q^2\right)\right]^2 \left(\frac{-d\omega}{\omega}\right) \\ &= \frac{3\alpha_s}{\pi} G(x, Q^2) \int_{\frac{x}{x_0}}^1 \omega^{\lambda-1} d\omega - \frac{81\alpha_s^2}{16R^2 Q^2} G^2(x, Q^2) \int_{\frac{x}{x_0}}^1 \omega^{2\lambda-1} d\omega. \end{aligned} \quad (2)$$

Let us consider a convenient variable  $t$  instead of  $Q^2$ , where,  $t = \ln(Q^2/\Lambda^2)$ ,  $\Lambda$  being the QCD cut off parameter. Then equation (2) can be rewritten as



$$\frac{dG(x,t)}{dt} = \frac{3\alpha_s}{\pi} G(x,t) \int_{x/x_0}^1 \omega^{\lambda-1} d\omega - \frac{81\alpha_s^2}{16R^2\Lambda^2 e^t} G^2(x,t) \int_{x/x_0}^1 \omega^{2\lambda-1} d\omega \quad (3)$$

Performing the integrations, equation (3) becomes

$$\frac{dG(x,t)}{dt} = \frac{3\alpha_s}{\pi} G(x,t) \frac{\left(1 - \left(\frac{x}{x_0}\right)^\lambda\right)}{\lambda} - \frac{81\alpha_s^2}{16R^2\Lambda^2 e^t} G^2(x,t) \frac{\left(1 - \left(\frac{x}{x_0}\right)^{2\lambda}\right)}{2\lambda}$$

$$\Rightarrow \frac{dG(x,t)}{dt} = H_1(x)G(x,t) - H_2(x)\frac{G^2(x,t)}{e^t} \quad (4)$$

$$\text{with, } H_1(x) = \frac{3\alpha_s}{\pi} \frac{\left(1 - \left(\frac{x}{x_0}\right)^\lambda\right)}{\lambda}$$

$$\text{and } H_2(x) = \frac{81\alpha_s^2}{32R^2\Lambda^2} \frac{\left(1 - \left(\frac{x}{x_0}\right)^{2\lambda}\right)}{\lambda}$$

Equation (4) can be solved as,

$$G(x,t) = \frac{(-1 + H_1(x))e^{H_1(x)t}}{H_2(x)e^{(-1+H_1(x))t} + C(-1 + H_1(x))} \quad (5)$$

where, C is a constant.

Now, at  $t = t_0$  for any lower value of  $Q = Q_0$  let us define

$$G_0 = G(x, t_0) = \frac{(-1 + H_1(x))e^{H_1(x)t_0}}{H_2(x)e^{(-1+H_1(x))t_0} + C(-1 + H_1(x))} \quad (6)$$

Thus, from equation (6) we get the value of C as

$$C = \frac{(-1 + H_1(x))e^{H_1(x)t_0} - H_2(x)e^{(-1+H_1(x))t_0}G_0}{G(-1 + H_1(x))} \quad (7)$$

Substituting C in equation (5) we get,

$$G(x, t) = \frac{(-1 + H_1(x))e^{H_1(x)t} G_0}{G_0 H_2(x) \left[ e^{(-1+H_1(x))t} - e^{(-1+H_1(x))t_0} \right] + (-1 + H_1(x))e^{H_1(x)t_0}} \quad (8)$$

Equation (8) gives the  $t$ - evolution of gluon distribution function  $G(x, Q^2)$ .

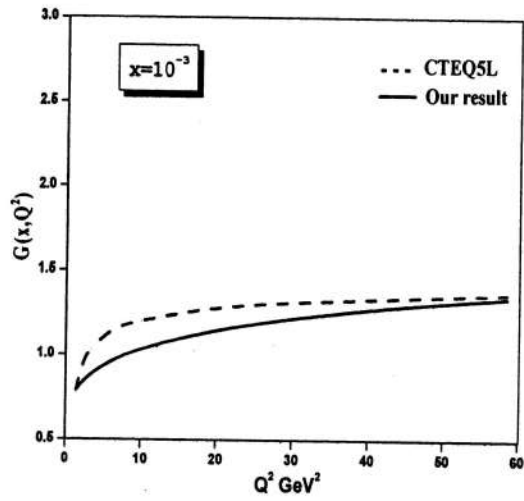
### Results and discussions:

We compared our results of  $t$ -evolution of gluon distribution function  $G(x, Q^2)$  with CTEQ 5L, CTEQ 6L and GRV 98 parametrizations. These parametrizations are different global QCD analysis of high energy interactions to determine the gluon and quark distributions in the nucleon. The CTEQ and GRV parametrizations presented a determination of the parton density from the H1 data at HERA. To compare our results with these parametrizations, we consider for CTEQ 5L the range  $1.43 \leq Q^2 \leq 100 \text{ GeV}^2$ , for CTEQ 6L the range  $3.4 \leq Q^2 \leq 152 \text{ GeV}^2$  and for GRV the range  $5 \leq Q^2 \leq 100 \text{ GeV}^2$ . In all graphs data points at lowest- $Q^2$  values are taken as inputs to test the  $t$ -evolution of gluon distribution function. The value of  $\Lambda$ , the QCD cut off parameter for both CTEQ (both CTEQ 5L and CTEQ 6L) and GRV is taken as 192 MeV. Here we have checked our results of  $t$ -evolution of  $G(x, Q^2)$  for  $x=10^{-3}$ ,  $10^{-4}$  and  $10^{-5}$  respectively. Figure 1(a), 1(b), 1(c) shows the comparison of our results of  $t$ -evolution of  $G(x, Q^2)$  with the CTEQ 5L parametrizations for  $x=10^{-3}$ ,  $10^{-4}$  and  $10^{-5}$  respectively. Figure 2(a), 2(b), 2(c) shows the comparison of our results of  $t$ -evolution of  $G(x, Q^2)$  with the CTEQ 6L parametrizations for  $x=10^{-3}$ ,  $10^{-4}$  and  $10^{-5}$  respectively. Similarly, Figure 3(a), 3(b), 3(c) shows the comparison of our results of  $t$ -evolution of  $G(x, Q^2)$  with the GRV parametrizations for  $x=10^{-3}$ ,  $10^{-4}$  and  $10^{-5}$  respectively. In the graphs the solid lines represent our result and dashed lines represent the results from CTEQ 5L parametrizations.

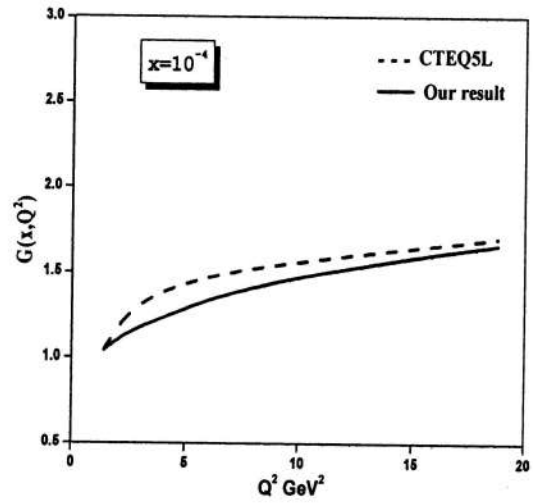
### Conclusion :

To conclude, we can say that our results of the solution of GLR equation show that the gluon distribution function increases with increasing  $Q^2$ . Results also confirm that the GLR equation has a tamed behavior with respect to the gluon saturation as  $Q^2$  increases and the obtained results at the hot spot are compatible with the results obtained by different parametrizations of parton distribution functions.

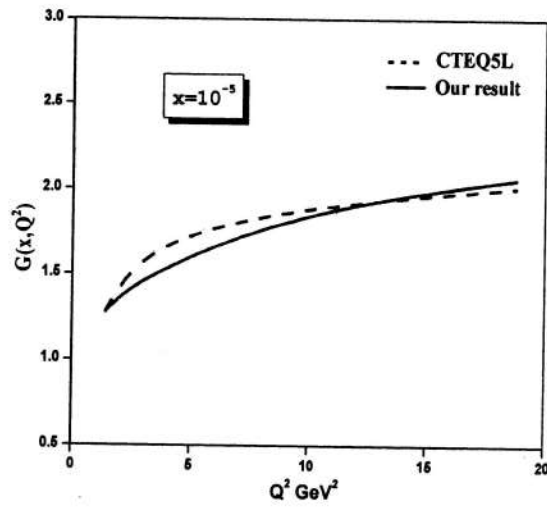
t- evolution results of GLR equation compared with CTEQ 5L parametrizations



1(a)



1(b)



1(c)

Fig 1(a,b,c): A plot of gluon distribution function  $G(x, Q^2)$  vs  $Q^2$  for  $x=10^{-3}$ ,  $10^{-4}$  and  $10^{-5}$  respectively compared with CTEQ 5L parametrizations. Here solid lines represent our result and dashed lines represent the results from CTEQ 5L parametrizations.

t- evolution results of GLR equation compared with CTEQ 6L parametrizations

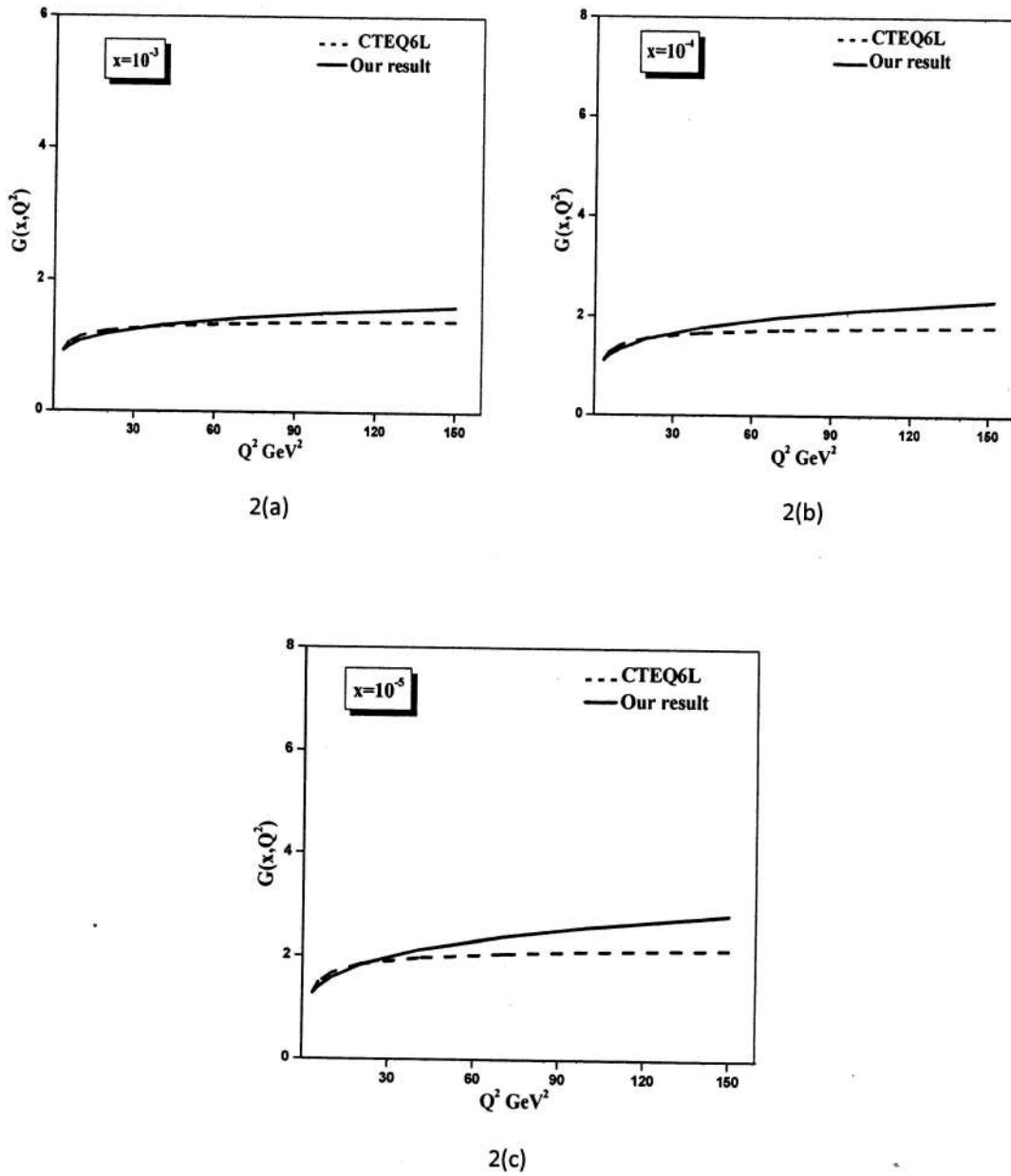
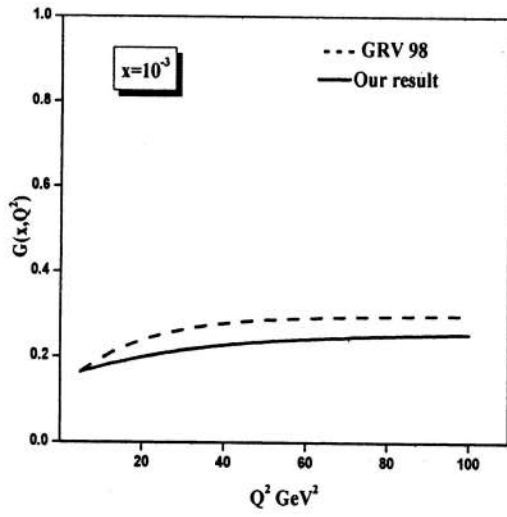
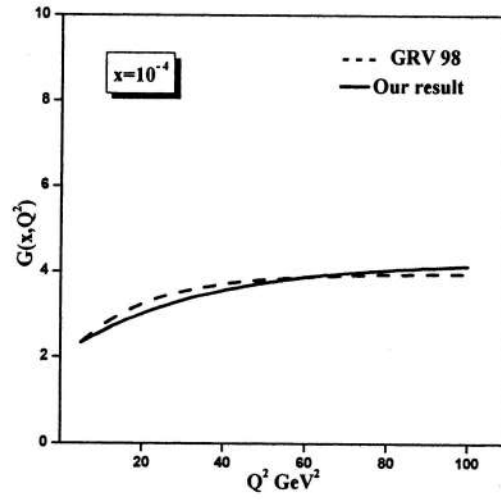


Fig 2 (a,b,c): A plot of gluon distribution function  $G(x, Q^2)$  vs  $Q^2$  for  $x=10^{-3}$ ,  $10^{-4}$  and  $10^{-5}$  respectively compared with CTEQ 6L parametrizations. Here solid lines represent our result and dashed lines represent the results from CTEQ 6L parametrizations.

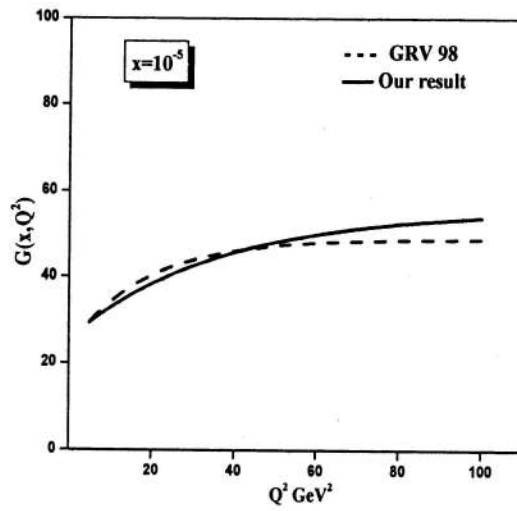
t- evolution results of GLR equation compared with GRV parametrizations



3(a)



3(b)



3(c)

Fig 3(a,b,c): A plot of gluon distribution function  $G(x, Q^2)$  vs  $Q^2$  for  $x=10^{-3}$ ,  $10^{-4}$  and  $10^{-5}$  respectively compared with GRV parametrizations. Here solid lines represent our result and dashed lines represent the results from GRV parametrizations.

### Participation in conferences:

The Project Fellow has attended the "DAE-BRNS workshop on Hadron Physics" held at Bhaba Atomic Research Center (BARC), Mumbai from October 31 to November 4, 2011.

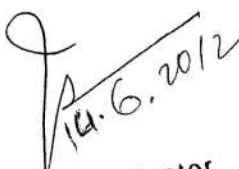
### Publications:

- (i) "Solution of singlet Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equation in next-to-next-to-leading order at small-x" M Devese, R Baishya and J K Sharma; Indian J Phys (February 2012) 86(2):141–144 (DOI 10.1007/s12648-012-0015-4)
- (ii) "Evolution of singlet structure functions from DGLAP equation at next-to-next-to-leading order at small-x" M Devese, R Baishya and J K Sharma (Accepted for publication in Eur. Phys. J C)

### References:

1. V N Gribov and L N Lipatov Sov. J. Nucl. Phys. **15** 438 (1972)
2. L N Lipatov Sov. J. Nucl. Phys. **20** 94 (1975)
3. S Moch and J A M Vermaseren Nucl. Phys. **B573** 853 (2000)
4. U Jamil and J K Sarma Indian J. Phys. **85** 141 (2011)
5. E. Ferreiro, E. Iancu, A. Leonidov and L. McLerran, Nucl. Phys. A701, 489 (2002)
6. W. Zhu and Z. Shen, arXiv:hep-ph/0406213v2, Mar, 2005
7. M. Gluck, E. Reya, and A. Vogt, Z. Phys. C 48 (1990) 471
8. M. Stramann and W. Vogelsang, Phys. Rev. D 64, 114007 (2001)
9. A. M. Cooper-Sarkar and R. C.E. D Evenish, Acta. Phys. Pol. B 34, 2911 (2003)
10. L. N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 338
11. E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP 45 (1977) 199
12. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 338
13. L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rep. 100 (1983) 1
14. M. Froissart, Phys. Rev. 123 (1961) 1053
15. A. Martin, Phys. Rev. 129(1963) 1432
16. A. Martin, Phys. Rev. 129 (1963) 143
17. H. Weigert, Nucl. Phys. A 703 (2002) 823
18. I. Balitsky, Nucl. Phys. B463 (1996) 99

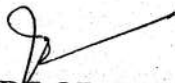
19. I. Balitsky, Phys. Rev. Lett. 81 (1998) 2024; Phys. Rev. D 60 (1999) 014020
20. Yu. V. Kovchegov, Phys. Rev. D 60 (1999) 034008
21. A. H. Mueller, Nucl. Phys. B 415 (1994) 373
22. E. Levin and K. Tuchin, Nucl. Phys. B 573 (2000) 833
23. J. Bartels, V. Fadin, L.N. Lipatov, Nucl. Phys. B 696 (2004) 255
24. A.H. Mueller and D.N. Triantafyllopoulos, Nucl. Phys. B 640 (2002) 331
25. A. M. Stasto, K. Golec-Biernat and J. Kwieciński, Phys. Rev. Lett. 86 (2001) 596
26. T. Gehrmann and E. Remiddi, Comput. Phys. Commun. 141 (2001) 296
27. M. R. Adams et. al., E665, Phys. Rev. D 54, 3006 (1996)
28. M. Arneodo et. al., CERN-NA-037, NMC, Nucl. Phys. B 483, 3 (1997)
29. W. Zhu and Z. Shen, arXiv:hep-ph/0406213v2, Mar, 2005
30. W. Furmanski and R. Petronzio, Phys. Lett. B 97, 437 (1980)
31. J. Ruan and W. Zhu, arXiv:hep-ph/0903.3073v1, Mar, 2009
32. A. Donnachie and P. V. Landshoff, Phys. Lett. B 550, 160 (2002)
33. R. D. Ball and P. V. Landshoff, hep-ph/9912445
34. P. Desgrolard, A. Lengyel, and E. Martynov, J. High Energy Phys. 0202, 029 (2002)

  
14.6.2012  
Principal Investigator  
UGC Major Project :  
Studies on Gluon Distribution Function  
and Recombination of partons  
TEZPUR UNIVERSITY

UNIVERSITY GRANTS COMMISSION  
BAHADUR SHAH ZAFAR MARG  
NEW DELHI – 110002

Utilization Certificate

Certified that the grant of Rs. NIL received during the year 2012-2013 from the University Grants Commission under the scheme of support for the Major Research Project entitled “Studies on Gluon Distribution Function and Recombination of Partons” vide UGC letter F. No. 37-369/2009 (ASS)(SR) dated 12 January, 2010 and Rs.74,456/- from account of unspent balance of second year 2011-2012, has been fully utilized for the purpose of project work in accordance with the terms and conditions laid down by the University Grants Commission. There is no unutilized balance.



SIGNATURE OF PRINCIPAL  
INVESTIGATOR

Principal Investigator  
UGC Major Project :  
Studies on Gluon Distribution Function  
and Recombination of partons  
TEZPUR UNIVERSITY



FINANCE OFFICER  
OSD Finance  
Tezpur University



REGISTRAR  
Registrar  
Tezpur University



**UNIVERSITY GRANTS COMMISSION**  
**BAHADUR SHAH ZAFAR MARG**  
**NEW DELHI – 110002**

**STATEMENT OF EXPENDITURE IN RESPECT OF MAJOR RESEARCH PROJECT**

1. Name of Principal Investigator: Prof. Jayanta Kumar Sarma
2. Department. Of University: Department of Physics, Tezpur University
3. UGC approval No. and date: F. No. 37-369/2009(ASS)(SR) dated 12 January, 2010
4. Title of the Research Project: "Studies on Gluon..... of Partons"
5. Effective date of starting the project: 16.07.2010 (Considering the date of joining of the project fellow)
6. Date of joining of the project fellow: 16.07.2010
7. Details of Expenditure:

Sl. No.	Item	Expenditure Incurred in 2010-11 (in Rs.)	Expenditure Incurred in 2011-12 (in Rs.)	Expenditure Incurred for the period 1 <sup>st</sup> April, 2012-31 <sup>st</sup> December, 2012 (in Rs.)
i.	Books	5154	14350	9744
ii.	Equipments	49825	---	---
iii.	Contingency	11651	19408	8838
iv.	Consumables	4662	9806	---
v.	Travel	5854	19195	17474
vi.	Staff Salary (@8600/-pm including HRA)	73,239	103200	34400
vii.	Overhead	15000	---	4000
	<b>Total</b>	<b>1,65,385</b>	<b>1,65,959</b>	<b>74,456</b>

8. Committed Fellowship to the project fellow with arrears: Rs. 2,81,109/-

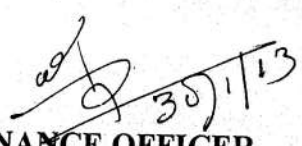
Refer  
Tezpur University  
Department  
Prof. D.K.  
To,  
Draft

It is certified that the appointment has been made in accordance with the terms and conditions laid down by the Commission.  
If as a result of check or audit, some irregularity is noticed, on later date, action will be taken to refund, adjust or regularize the objected amounts.  
3. Payment @ revised rates shall be made with arrears on the availability of additional funds.  
4. Certified that the grant of **Rs. NIL** received during the year **2012-2013** from the University Grants Commission under the scheme of support for the Major Research Project entitled "Studies on Gluon Distribution Function and Recombination of Partons" vide UGC letter F. No. 37-369/2009 (ASS)(SR) dated 12 January, 2010 and Rs.74,456/- from account of unspent balance of second year 2011-2012, has been fully utilized for the purpose of project work in accordance with the terms and conditions laid down by the University Grants Commission. There is no unutilized balance.



**SIGNATURE OF PRINCIPAL INVESTIGATOR**

**Principal Investigator  
UGC Major Project :  
Studies on Gluon Distribution Function  
and Recombination of partons  
TEZPUR UNIVERSITY**



**FINANCE OFFICER**  
*CSD Finance*  
*Tezpur University*




**REGISTRAR**  
*Registrar*  
*Tezpur University*

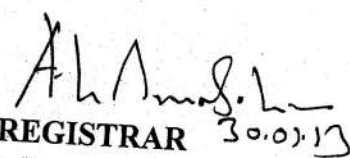
**DUE AND DRAWN STATEMENT OF FELLOWSHIP**

1. Name of the project fellow : MISS MAYURI DEVEE
2. Date of joining of the project fellow: 16/07/2010

Period	Amount of fellowship payable (in Rs.) (A)	Amount of fellowship drawn (in Rs.) (B)	Amount of fellowship due (in Rs.) (C=A-B)
16/07/2010 – 31/03/2011	1,31,148 (@ Rs.15,400/- p.m. including HRA @10%)	73,239 (@ Rs.8,600/- p.m. including HRA @7.5%)	57,909
01/04/2011 – 31/03/2012	1,84,800 (@ Rs.15,400/- p.m. including HRA @10%)	1,03,200 (@ Rs.8,600/- p.m. including HRA @7.5%)	81,600
01/04/2012 – 31/01/2013	1,76,000 (@ Rs.17,600/- p.m. including HRA @10%)	34,400 (@ Rs.8,600/- p.m. including HRA @ 7.5%)	1,41,600
<b>TOTAL</b>			<b>2,81,109</b>

  
**SIGNATURE OF PRINCIPAL INVESTIGATOR**  
Principal Investigator  
UGC Major Project :  
Studies on Gluon Distribution Function  
and Recombination of partons  
TEZPUR UNIVERSITY

  
**FINANCE OFFICER**  
OSD Finance  
Tezpur University

  
**REGISTRAR** 30.01.13  
Registrar  
Tezpur University