

UNIVERSITY GRANTS COMMISSION

BAHADUR SHAH ZAFAR MARG

NEW DELHI – 110 002

**PROFORMA FOR SUBMISSION OF INFORMATION AT THE TIME OF SENDING THE
FINAL REPORT OF THE WORK DONE ON THE PROJECT**

1. Title of the Project:

A study of k-hyponormality for the finite rank perturbation of weighted shift operators

2. NAME AND ADDRESS OF THE PRINCIPAL INVESTIGATOR

Munmun Hazarika

Department of Mathematical Sciences, Tezpur University

3. NAME AND ADDRESS OF THE INSTITUTION Tezpur University

4. UGC APPROVAL LETTER NO. AND DATE: F. No. 42-1000/2013 (SR), 25 March, 2013

5. DATE OF IMPLEMENTATION 01.04.2013

6. TENURE OF THE PROJECT 2 years

7. TOTAL GRANT ALLOCATED Rs.1,45,000/-

8. TOTAL GRANT RECEIVED Rs.1,45,000/-

9. FINAL EXPENDITURE Rs.1,35,572/-

10. TITLE OF THE PROJECT A study of k-hyponormality for the finite rank perturbation of weighted shift operators

11. OBJECTIVES OF THE PROJECT: To develop the notion of perturbation of weighted shift operators, particularly in reference to the concepts of k-hyponormality and weak k-hyponormality. This will help to bridge the gap between subnormality and hyponormality.

12. WHETHER OBJECTIVES WERE ACHIEVED: Yes, the outcome was satisfactory and the objectives were achieved. The results derived therein enhances our understanding of the relation between subnormality and hyponormality.

13. ACHIEVEMENTS FROM THE PROJECT: There already exists in the literature, different necessary and sufficient conditions for a weighted shift operator to be either hyponormal, or weakly hyponormal, or 2-hyponormal, or quadratic hyponormal, or subnormal. However, these necessary and sufficient conditions are all framed in terms of the "weight sequence" of the particular weighted shift. This immediately implies that any change or perturbation in the weights could reflect upon the hyponormality or any other similar property of the weighted shift. This aspect has been investigated in detail in the present work. Conditions have been framed here which can exhaustively determine the situations where a perturbed shift will still retain its original property of hyponormality/ 2-hyponormality/ quadratic hyponormality/ subnormality.

14. SUMMARY OF THE FINDINGS: In the work we consider the weighted shift operator W_α with scalar weight sequence $\{\alpha_n\}$. Also for $k \geq 1$ and $j \geq 0$, let $\alpha[j:x]$ denote the sequence $\{\alpha_0, \dots, \alpha_{j-1}, x, \alpha_{j+1}, \dots\}$ where x is a scalar. Also $\Omega_\alpha(k, j) := \{x: W_{\alpha[j:x]} \text{ is } k\text{-hyponormal}\}$ and $\omega_\alpha(k, j) := \{x: W_{\alpha[j:x]} \text{ is weakly } k\text{-hyponormal}\}$.

➤ It is shown that rank-one perturbations of k -hyponormal weighted shifts which preserve k -hyponormality form a convex set. That is, if W_α is k -hyponormal then $\Omega_\alpha(k, j)$ is a convex set. It is also shown that if W_α is weakly k -hyponormal, then $\omega_\alpha(k, j)$ is a convex set.

➤ For the weights $\alpha_n = \sqrt{\frac{n+1}{n+2}}$ for all n , we have proved the following :

- For $x \in [k_1, k_2]$, the weighted shift $W_{[(0:x), (1:x)]}$ is polynomially quadratically hyponormal (p.q.h.), where $k_1 = 0.630435$, and $k_2 = 0.737144$.
- For $y \in [k_1, k_2]$, $\{x: W_{[(0:x), (1:y)]} \text{ is p. q. h.}\} = (0, y]$.
- If either $y < k_1$ or $y > k_2$, then there exists $0 < x \leq y$ such that $W_{[(0:x), (1:y)]}$ is not p.q.h.

➤ We establish a set of sufficient conditions under which there exists $\varepsilon > 0$ such that for $x \in (\alpha_j - \varepsilon, \alpha_j + \varepsilon)$, $W_{\alpha[j:x]}$ will again be 2-hyponormal. Applying these conditions we can completely determine the situations where 2-hyponormality preserving perturbations do not exist.

- It is well known that W_α is hyponormal if and only if $\alpha_n \leq \alpha_{n+1}$ for all n . From this it is obvious that for a strictly increasing weight sequence, any slight perturbation of the i^{th} weight still retains the hyponormality property of the perturbed shift. However, it is surprising and interesting that the same is not true for a two variable weighted shift as shown in our work here. In fact we have framed a set of positivity conditions which can completely determine hyponormality of the perturbed shift. Moreover, even if the perturbed shift is not hyponormal, we have shown that it will still remain weakly hyponormal for sufficiently small perturbations.

These results have been published in research articles listed in point no. 17 below.

15. CONTRIBUTION TO THE SOCIETY : The outcome of the project and the results derived therein will help further the knowledge of the subject and also serve as a platform for further research.

16. WHETHER ANY PH.D. ENROLLED/PRODUCED OUT OF THE PROJECT: There was no provision for research scholar in the project. However, Mr. Bimalendu Kalita, a research scholar under my supervision have worked extensively along with me during his Ph. D. work in a similar topic.

17. NO. OF PUBLICATIONS OUT OF THE PROJECT : 03

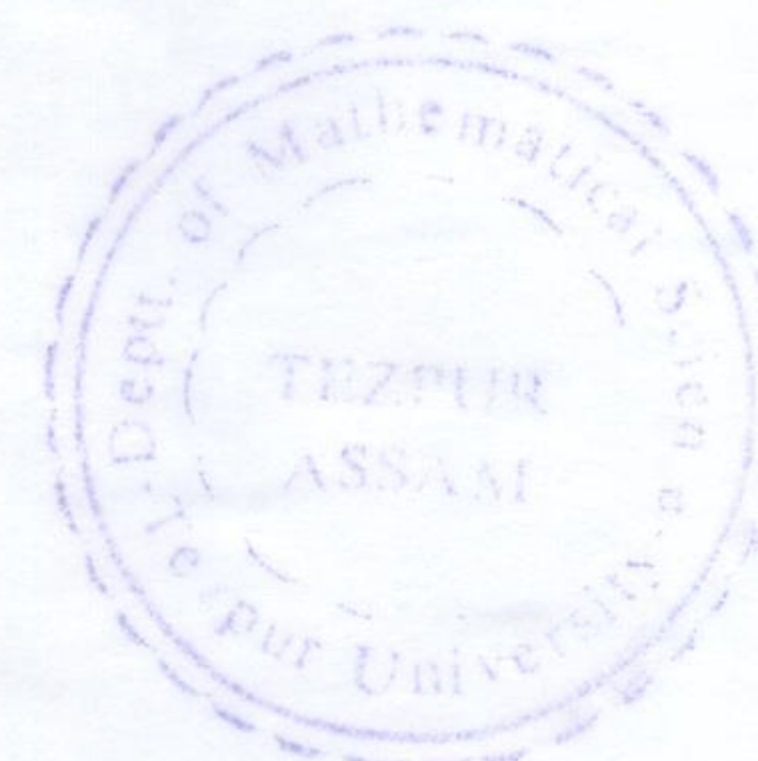
- Munmun Hazarika and B. Kalita "Back-step extension of weighted shifts" in *Bull. Cal. Math. Soc.*, Vol.106(3), 169-188, **2014**.
- Munmun Hazarika and B. Kalita "Convexity of the region of perturbation of a weakly k-hyponormal weighted shift" in *International Journal of Functional Analysis, Operator Theory and Applications*, Vol.5(3), 141-151, **2013**.
- Munmun Hazarika and B. Kalita "Perturbation of two hyponormal weighted shifts" in *Bull. Cal. Math. Soc.*, Vol.105(4), 269-284, **2013**.

(MUNMUN HAZARIKA)

(PRINCIPAL INVESTIGATOR)

(PRINCIPAL)

Munmun Hazarika
15/9/16



(Seal)

UNIVERSITY GRANTS COMMISSION
BAHADUR SHAH ZAFAR MARG
NEW DELHI - 110 002

STATEMENT OF EXPENDITURE IN RESPECT OF MINOR
RESEARCH PROJECT

1. Name of Principal Investigator: Dr. Munmun Hazarika
2. Deptt. of University/College: Mathematical Sciences, Tezpur University
3. UGC approval No. and Date: F. No. 42-1000/2013 (SR), 25 March, 2013
4. Title of the Research Project: A study of k-hyponormality for the finite rank perturbation of weighted shift operators
5. Effective date of starting the project: 1st April, 2013
6. a. Period of Expenditure: From 1 April, 2013 to 31 March, 2015
- b. Details of Expenditure

S. No.	Item	Amount Approved Rs	Expenditure Incurred Rs
i.	Books and Journals	40,000/-	2740/-
ii.	Equipment (Please enclose quotation)	75,000/-	73400/-
iii.	Contingency	20,000/-	19258/-
iv.	Field work/Travel (Give details in the proforma at Annexure-VI)	40,000/-	40129/-

7. If as a result of check or audit objective, some irregularity is noticed, later date, action will be taken to refund, adjust or regularize the objected amounts.


Finance Officer
Tezpur University

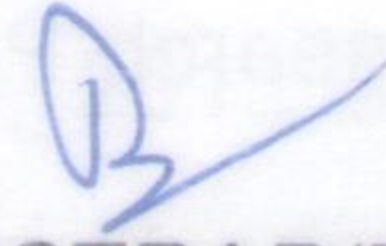
8. It is certified that from the grant of Rs. 145,000/- (*Rupees One Lakh Forty Five Thousand only*) received from the University Grants Commission under the scheme of support for Major Research Project entitled:

A study of k-hyponormality for the finite rank perturbation of weighted shift operators vide UGC letter No. F. No. 42-1000/2013 (SR), 25 March, 2013, an amount of Rs 1,35,527/- (*Rupees One Lakh Thirty five Thousand Five Hundred Twenty seven only*) has been utilized for the purpose for which it was sanctioned and in accordance with the terms and conditions laid down by the University Grants Commission.

The remaining amount of Rs 9,473/- plus the interest Rs 596/- , totaling to an amount of **Rs10,069/-** (*Rupees ten thousand sixty nine only*) is returned to UGC.

Hazarika 19/9/16

SIGNATURE OF PRINCIPAL
INVESTIGATOR



REGISTRAR/PRINCIPAL

(Seal)

Registrar

Tezpur University

[Faint signature]
Finance Officer
Tezpur University

**UNIVERSITY GRANTS COMMISSION
BAHADUR SHAH ZAFAR MARG
NEW DELHI – 110 002**

STATEMENT OF EXPENDITURE INCURRED ON FIELD WORK

Name of the Principal Investigator: Dr. Munmun Hazarika

Name of place visited	Duration of visit		Mode of Journey	Expenditure Incurred (Rs.)
	From	To		
Delhi University	20.04.13	24.04.13	Air Travel	Rs. 19,400/-
I.I.Sc. Bangalore	15.12.13	21.12.13	Air Travel	Rs. 20,729/-

Certified that the above expenditure is in accordance with the UGC norms for Major Research Projects

Munmun Hazarika 19/9/16
SIGNATURE OF PRINCIPAL
INVESTIGATOR

B
REGISTRAR/PRINCIPAL
Registrar
Tezpur University

UNIVERSITY GRANTS COMMISSION
BAHADUR SHAH ZAFAR MARG
NEW DELHI - 110 002

UTILIZATION CERTIFICATE

It is certified that from the grant of Rs. 145,000/- (*Rupees One Lakh forty five Thousand only*) received from the University Grants Commission under the scheme of support for Major Research Project entitled: *A study of k-hyponormality for the finite rank perturbation of weighted shift operators*, vide UGC letter No. F. No. 42-1000/2013 (SR), 25 March, 2013 an amount of Rs 1,35,527/- (*Rupees One Lakh Thirty five Thousand Five Hundred Twenty Seven only*) has been utilized for the purpose for which it was sanctioned and in accordance with the terms and conditions laid down by the University Grants Commission.

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vide RTGS No: SBIN616275148063 dtd. 01-10-2016

M. Lazarika 19/9/16
SIGNATURE OF PRINCIPAL
INVESTIGATOR

D3
REGISTRAR/PRINCIPAL
(SEAL)
Registrar
Tezpur University

STATUTORY
AUDITOR
(SEAL)